The structure of neutron deficient Sn isotopes

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February 9, 2008

Abstract

In this work we present a realistic shell model calculation of the low-lying spectra for Sn isotopes with mass number $A = 102 - 108$. The effective shell model interaction was derived from a realistic nucleon-nucleon interaction employing perturbative many-body techniques.

1 Introduction

During the last few years through the radioactive nuclear beams program a rich variety of data has become available for nuclei far from the stability line. Recently, substantial progress has been made in the spectroscopic approach to the neutron deficient doubly magic ^{100}Sn core. Experimental spectroscopic data are presently available down to ^{104}Sn [1-4]. Thus theoretical shell model calculations for these nuclei are greatly needed. The starting point for such an analysis is the doubly magic core ^{100}Sn. This system has been studied theoretically, using either approaches inspired by the relativistic Serot-Walecka [5] model, such as the calculations of Hirata *et al.* [6] and Nikolaus *et al.* [7], or non-relativistic models used by Leander *et al.* [8]. In both approaches it is concluded that ^{100}Sn is a reasonably stable closed core. Thus the Sn isotopes should be well suited for a shell model analysis.

From a theoretical point of view the Sn isotopes are interesting to study as they are an unique testing ground for nuclear structure calculations. We have a large sequence of nuclei which can be described in terms of the neutron degrees of freedom only. Here, it is often assumed that the seniority or generalized seniority scheme is a good approximation [9] to the large shell model problem these isotopes represents. This can now be tested with our present shell model code which is able to handle many of the isotopes. Also the question of an additional effective three-body interaction [10] may be investigated when the number of valence neutrons becomes significant.

In several papers [1-3] Schubart *et al.* have calculated energy spectra above ^{100}Sn within the framework of the spherical shell model. However, their results have some deficiencies. The two-body interaction they used had to be scaled by a factor 1.4 in

order to give a reasonable agreement with experiment, and the E2 transition rates measured in ^{104}Sn and ^{106}Sn are not well understood in their model.

A realistic shell model calculation for even Sn isotopes with mass number $A =$ 102, 104 and 106 was recently published by us in ref. [11].In this work we present a calculation for $^{102-108}$Sn, including the odd isotopes, using the doubly magic ^{100}Sn as core.

In the next section we briefly outline the derivation of the effective interaction and some detailes for the shell model calculation. Our results are presented in sect. 3, where the energy spectra and also some aspects of the single particle energies are discussed.

2 Effective interaction and shell model calculation

In this work we derive a two-body neutron-neutron effective interaction calculated from conventional perturbative many-body techniques [12]. Here we sketch briefly the main ingredients.

The many-body Schrödinger equation for an A-nucleon system is given by

$$H\Psi_i(1,...,A) = E_i\Psi_i(1,...,A) \tag{1}$$

with $H = T + V$, T being the kinetic energy operator and V the nucleon-nucleon potential. For the nucleon-nucleon potential V, we adopt here the parameters of the Bonn A potential defined in table A.2 of ref. [13]. E_i and Ψ_i are the eigenvalues and eigenfunctions for a state i. Introducing the auxiliary single-particle potential U, H can be rewritten as

$$H = H_0 + H_1, \quad H_0 = T + U, \quad H_1 = V - U \ . \tag{2}$$

If U is chosen such that H_1 becomes small, then H_1 can be treated as a perturbation. The eigenfunctions of H_0 are then the unperturbed wave functions $|\psi_i\rangle$ with W_i as the respective eigenvalue.

Usually, one is only interested in solving the Schrödinger equation for certain low-lying states. It is then customary to divide the Hilbert space into a model space defined by the operator P

$$P = \sum_{i=1}^{d} |\psi_i\rangle\langle\psi_i| \tag{3}$$

with d being the size of the model space, and an excluded space defined by the operator Q

$$Q = \sum_{i=d+1}^{\infty} |\psi_i\rangle\langle\psi_i| \tag{4}$$

such that $PQ = 0$. In this work we define the model space to consist of the orbitals in the $N = 4$ oscillator shell ($1d_{5/2}$, $0g_{7/2}$, $1d_{3/2}$, $2s_{1/2}$) and the intruder orbital $0h_{11/2}$

from the $N = 5$ oscillator shell. As we show below, the inclusion of the $h_{11/2}$ orbit to our model space is needed in order to obtain a quantitative description of the data.

No experimental information is available for the ^{101}Sn one-neutron system to establish the single-particle energies necessary for the shell model calculation. Thus, these must be evaluated theoretically. In the present calculation the data are taken from ref. [3], with $\varepsilon_{d_{3/2}} - \varepsilon_{d_{5/2}} = 2.89$ MeV, $\varepsilon_{s_{1/2}} - \varepsilon_{d_{5/2}} = 2.80$ MeV and $\varepsilon_{h_{11/2}} - \varepsilon_{d_{5/2}}$ set to 5.0 MeV.

Eq. (1) can be rewritten as

$$PH_{\text{eff}}P\Psi_i = P(H_0 + V_{\text{eff}})P\Psi_i = E_i P\Psi_i \tag{5}$$

where H_{eff} is an effective Hamiltonian acting solely within the chosen model space. In this work we have used an iterative approach to the energy-independent expansion for H_{eff} advocated by Lee and Suzuki (LS) [14].The LS expansion for the effective interaction is then formally given as

$$H_{\text{eff}} = H_0 + \lim_{n\to\infty} R_n, \tag{6}$$

with

$$R_n = \left[1 - \hat{Q}_1 - \sum_{m=2}^{n} \hat{Q}_m \prod_{k=n-m+1}^{n-1} R_k\right]^{-1} \hat{Q}. \tag{7}$$

Here we have defined $\hat{Q}_m = \frac{1}{m!}\frac{d^m\hat{Q}}{d\Omega^m}$, where Ω is the energy of the interacting nucleons in a medium. To define the $\hat{Q}$-box, we include all diagrams through third order in the interaction, as defined in the appendix of ref. [15]. The $\hat{Q}$-box consists of all valence-linked, non-folded and irreducible diagrams, and is conventionally defined in terms of the G-matrix

$$G = V + V\frac{Q}{\Omega - QH_0Q}G. \tag{8}$$

We solve eq. (8) through the so-called double-partitioning scheme defined in ref. [16]. The operator Q is constructed so as to prevent scattering into intermediate states with a nucleon in any of the states defined by the orbitals from the $1s_{1/2}$ to the $0g_{9/2}$ states or with two nucleons in the sdg-shell ($0g_{9/2}$ excluded) or two nucleons in the pfh-shell. A harmonic-oscillator basis was chosen for the single-particle wave functions, with an oscillator energy $\hbar\omega$, ω being the oscillator frequency.

The shell model calculation for the Sn isotopes are based on eq. (5). It is important for the present work to include all neutron configurations in the model space. This means diagonalization of large energy matrices. We have chosen a procedure based on the Lanczos algoritm ref. [17] where the states are given in the m-scheme Slater determinant basis. The dimensionality of the matrices for different isotopes are shown in table 1. This technique is an iterative procedure where the lowest states are first calculated. At present ^{108}Sn is not completed. Only the four lowest states are found (see fig. 2).

Table 1: Dimension of the basis set in calculations of $^{102-108}$Sn.

Isotope	^{102}Sn	^{103}Sn	^{104}Sn	^{105}Sn	^{106}Sn	^{107}Sn	^{108}Sn
Dimension	36	245	1 504	7 451	31 124	108 297	323 682

3 Results and discussion

At present rather few energy levels are known for the lightest Sn isotopes, a fact which makes a detailed comparison difficult. However, some characteristic properties can be found in the data. In the even isotopes there is a rather constant spacing between the 0^+ ground state and the first excited 2^+ state. Furthermore, in the odd systems a similar feature is found between the $5/2^+$ ground state and the first excited $7/2^+$ state up to ^{109}Sn. In ^{111}Sn these levels are interchanged, with the $7/2^+$ state as the ground state. This is well described in the BCS model originally analysed by Kisslinger and Sorensen [18] and also in the generalized seniority scheme discussed by Talmi [9].

In our extended shell model calculation two elements are important for the theoretical energy spectrum – the single-particle spectrum and the effective two-body matrix elements. We are reasonably confident about the calculation of the effective force. Similar methods have been used in the oxygen and calcium regions with reasonable results. However, the details of the single-particle spectrum are rather uncertain. In order to get a better understanding of the role of the single-particle energies we have investigated a simplified model. The model space is reduced to the $g_{7/2}$ and $d_{5/2}$ orbitals only, and the relative spacing has been varied. The results of two extreme cases, one with the two orbitals degenerated and one with the two orbitals separated by 1.0 MeV, are displayed in fig. 1. With this reduced model space the energy spectra for the sequence of even Sn isotopes with mass number A = 102 – 112 are calculated. When the $g_{7/2}$ and $d_{5/2}$ orbitals become too strongly separated, the $0^+ - 2^+$ spacing does not remain constant. The energy gap increases dramatically for ^{106}Sn, where the $d_{5/2}$ orbital is nearly filled. Thus the nearly degenerate values for the single-particle energies employed in ref. [3] seem reasonable and are used in our calculation. The calculated values of the higher single-particle levels have been used without further discussion.

The resulting energy spectra obtained with a complete single-particle basis including the $h_{11/2}$ orbital and the above-mentioned effective interaction are displayed in fig. 2 and fig. 3. For the even Sn isotopes (fig. 2) the excitation energies of the lowest $J = 2$ states are in good agreement with the experimental values, with a spacing of about 1.2 MeV. The theoretical spectra above the first excited $J = 2$ state are far more difficult to interpret. Compared to the levels experimentally found our calculation gives many more levels in the region between 1 and 2 MeV. However, we believe

that there are still more experimental levels to be found in this region in the future.

The odd Sn isotopes are displayed in fig. 3. The two lowest energy levels $J = 5/2^+$ and $J = 7/2^+$ are reproduced in correct order, but in ^{107}Sn the theoretical spacing is smaller than the experimental value. This may in turn lead to a $J = 7/2^+$ ground state in ^{109}Sn, contrary to what is experimentally found. Moreover, this could indicate that the higher single-particle levels should be lowered in order to reduce the filling of the $d_{5/2}$ and the $g_{7/2}$ levels. As in the even isotopes many more energy levels are calculated than known experimentally.

A further test of our results and the properties of the model wave function may be found in the two known $6^+ \rightarrow 4^+$ E2 transitions [3]. There are two measured E2 transitions, one in ^{104}Sn $6^+ \rightarrow 4^+$ with a strength of 4.0 Wu and one in ^{104}Sn $6^+ \rightarrow 4^+$ with strength 2.5 Wu. With the assumption that the observed 4^+ and 6^+ states are the lowest ones, our model is not able to reproduce the E2 transitions with sufficient strength. An alternative interpretation may be that the E2 transitions observed are between higher-lying states. We have calculated two $6^+ \rightarrow 4^+$ E2 transitions with strengths 2.1 Wu and 3.0 Wu, which are comparable to the observations in ^{104}Sn and ^{106}Sn, respectively. The transitions are calculated with an effective neutron charge $e_n = 1.0e$, a value which is within the range of theoretically calculated effective charges for the Sn isotopes [20]. Based on these values we may interpret our results in the following way: In ^{104}Sn the 4^+ state at 1.942 MeV corresponds to our 4_2^+ state and the 6^+ state at 2.257 MeV to our 6_2^+ state. Similarily in ^{106}Sn the 4^+ state at 2.017 MeV corresponds to our 4_3^+ state and the 6^+ state at 2.321 MeV to our 6_2^+ state.

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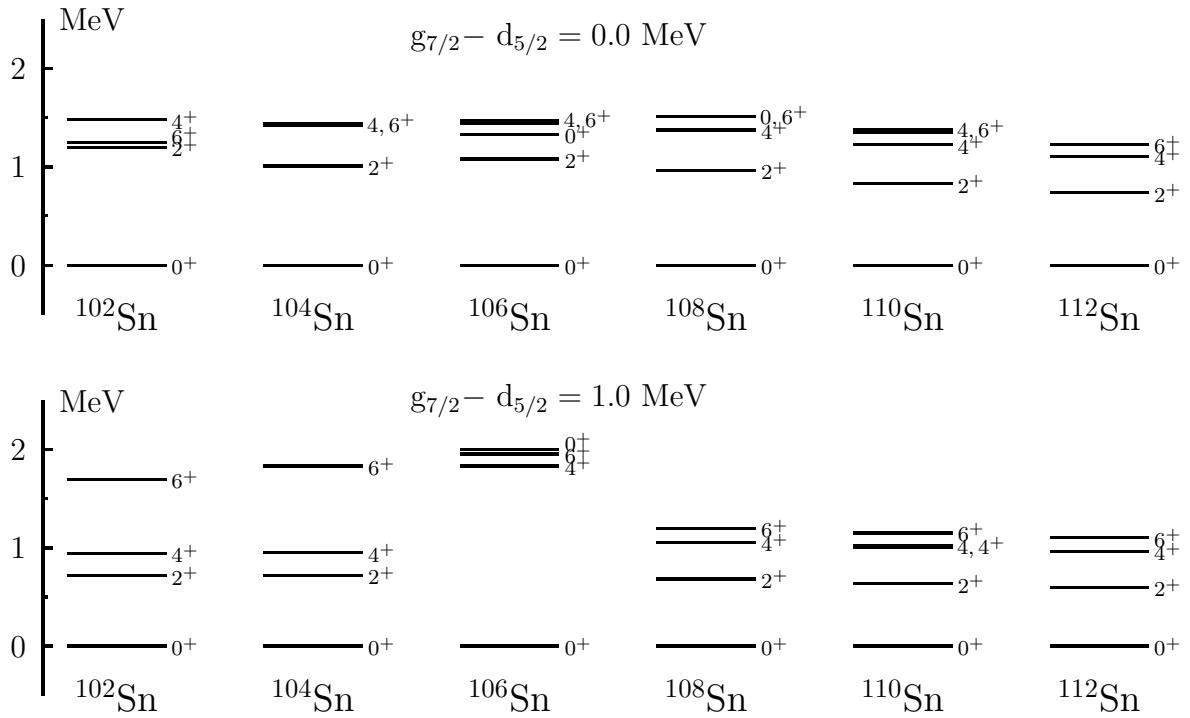

Figure 1: Energy spectra of $^{102-112}$Sn with a model space consisting of the $g_{7/2}$ and $d_{5/2}$ single-particle orbitals.

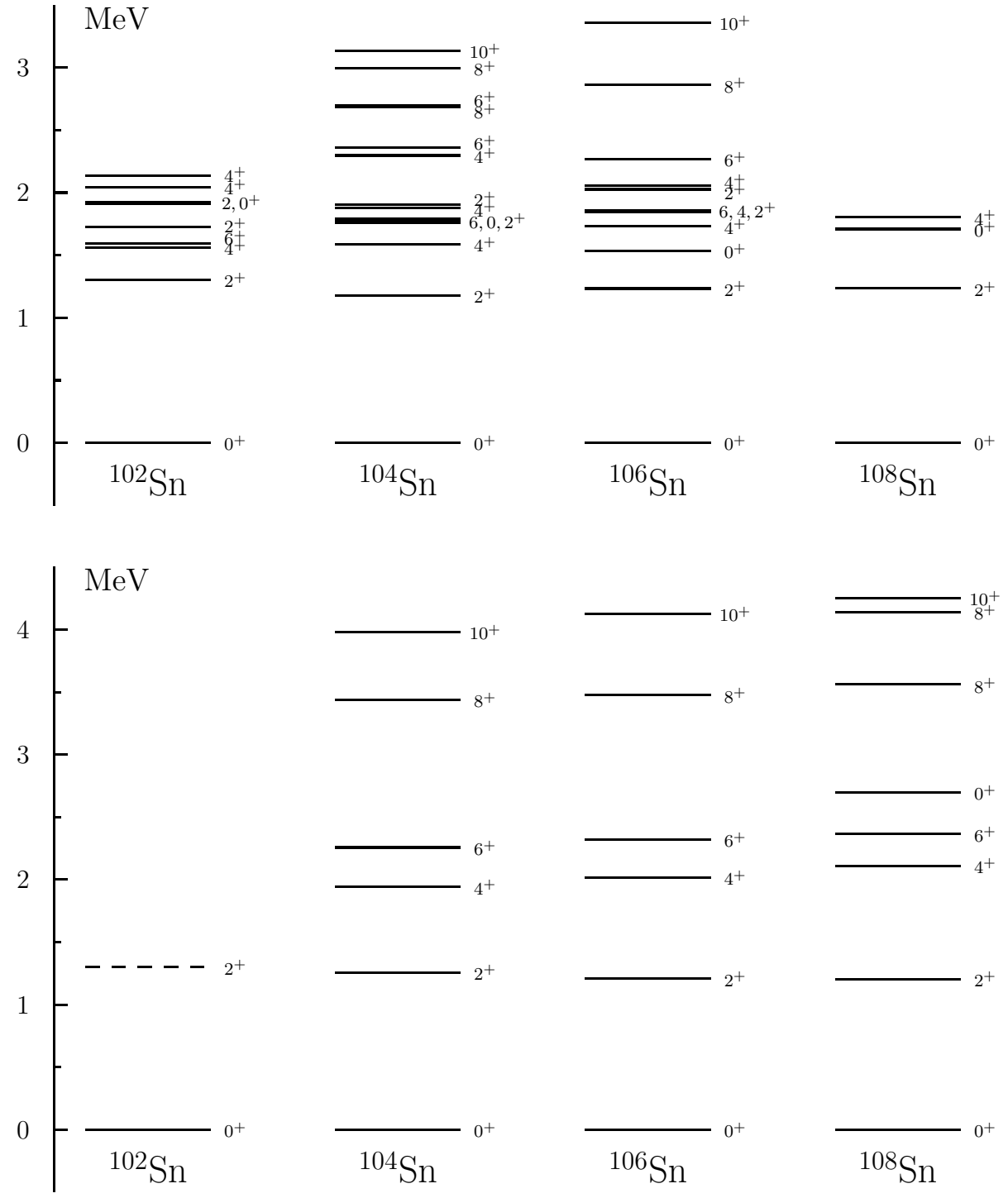

Figure 2: Theoretical (upper) and experimental (lower) energy spectra for the even isotopes $^{102-108}$Sn. The experimental 2^+ level (dashed) in ^{102}Sn is tentative [19].

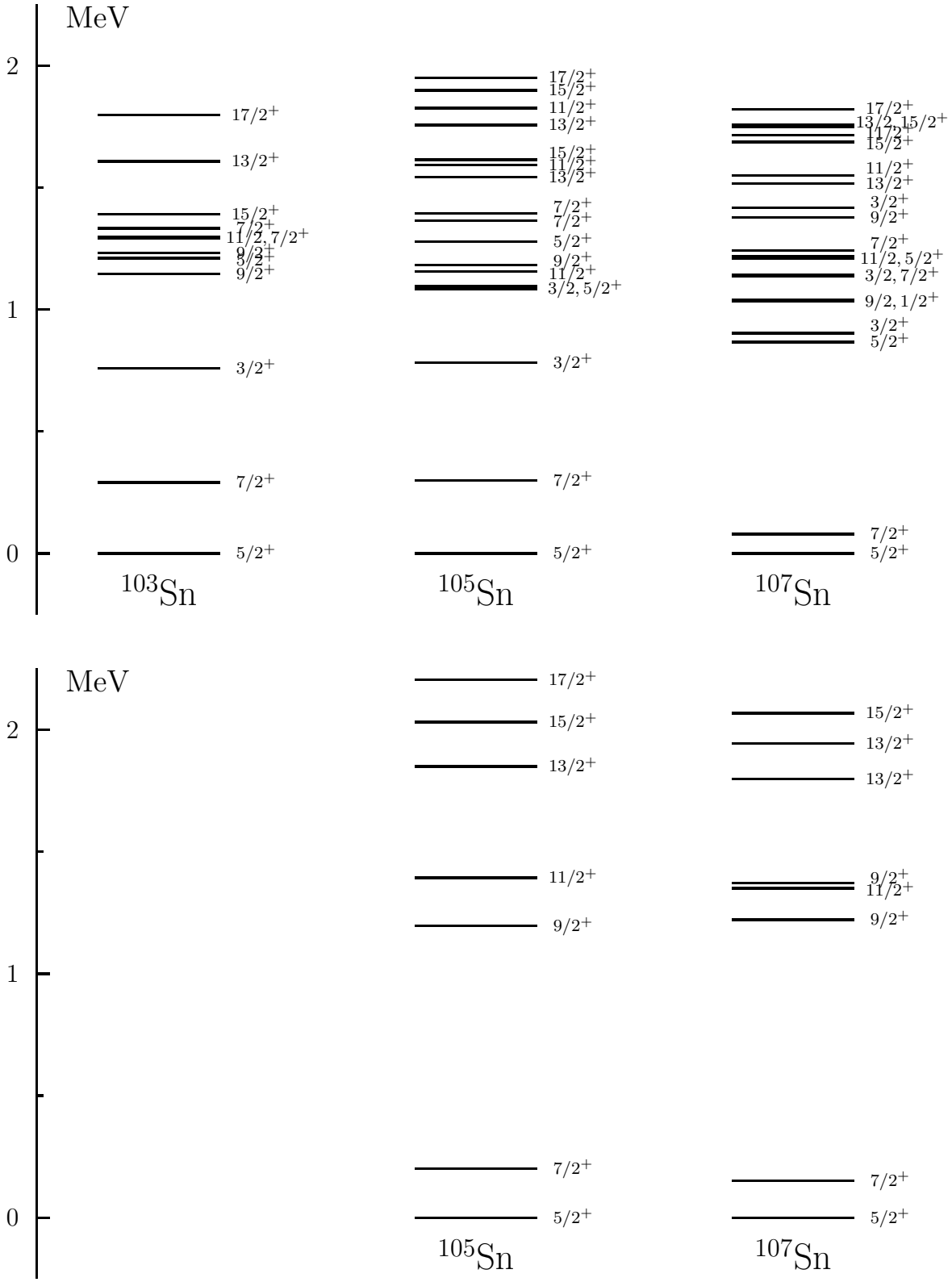

Figure 3: Theoretical (upper) and experimental (lower) energy spectra for the odd isotopes $^{103-107}$Sn.